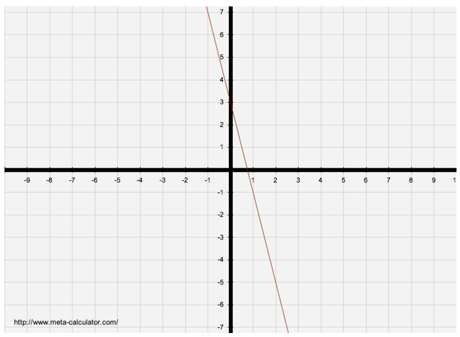
Functions

1. If $f(x) = x^2 + 4x - 3$ find $f(1), f(-1), f(0), f(\sqrt{2})$

$$f(1) = (1)^3 + 4(1) - 3 \Rightarrow f(1) = 1 + 4 - 3 \Rightarrow f(1) = 2$$
$$f(-1) = (-1)^3 + 4(-1) - 3 \Rightarrow f(-1) = -1 - 4 - 3 \Rightarrow f(-1) = -8$$
$$f(0) = (0)^3 + 4(0) - 3 \Rightarrow f(0) = -3$$
$$f(\sqrt{2}) = (\sqrt{2})^3 + 4(\sqrt{2}) - 3 \Rightarrow f(\sqrt{2}) = 2\sqrt{2} + 4\sqrt{2} - 3 \Rightarrow f(\sqrt{2}) = 6\sqrt{2} - 3$$

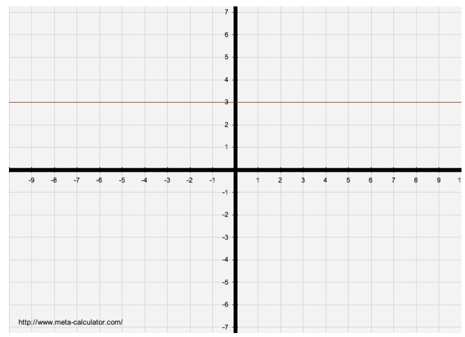
2. Sketch the graph and determine the domain and range of f

a) $f(x) = -4x + 3$



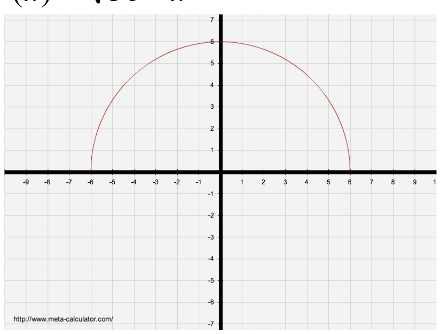
$D: (-\infty, \infty), R: (-\infty, \infty)$

b) $f(x) = 3$



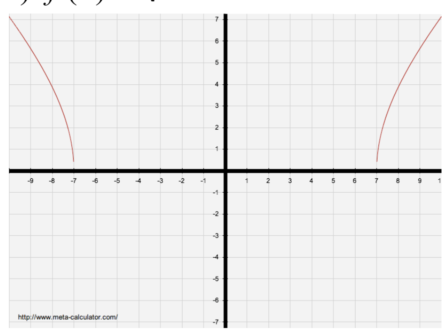
$D: (-\infty, \infty), R: [3]$

c) $f(x) = \sqrt{36 - x^2}$



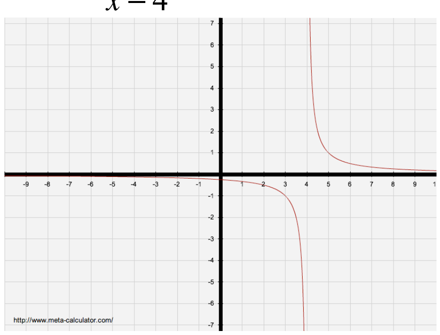
$D: [-6, 6], R: [0, 6]$

d) $f(x) = \sqrt{x^2 - 49}$



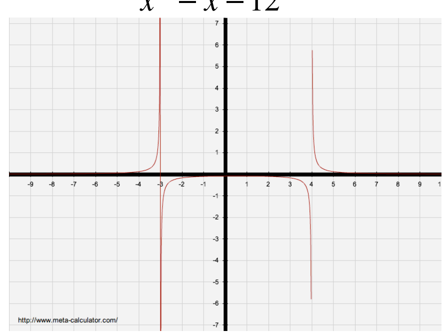
$D: (-\infty, -7] \cup [7, \infty), R: [0, \infty)$

e) $f(x) = \dfrac{1}{x - 4}$



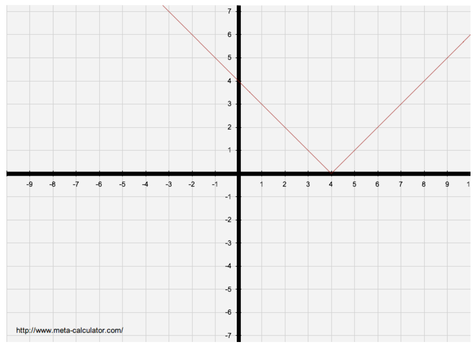
$D: (-\infty, 4) \cup (4, \infty), R: (-\infty, 0) \cup (0, \infty)$

f) $f(x) = \dfrac{5}{x^2 - x - 12}$



$D: (-\infty, -3) \cup (-3, 4) \cup (4, \infty), R: (-\infty, 0) \cup (0, \infty)$

g) $f(x) = |x-4|$



$D:(-\infty,\infty), R:[0,\infty)$

h) $f(x) = 2-\sqrt{x}$

$D:[0,\infty), R:(-\infty,2]$

3. Find the sum, difference and product of f and g

a)
$$f(x) = 3x^2, g(x) = \frac{1}{2x-3}$$
$$f(x)+g(x) = 3x^2 + \frac{1}{2x-3} \Rightarrow \frac{6x^3-9x^2+1}{2x-3}$$
$$f(x)-g(x) = 3x^2 - \frac{1}{2x-3} \Rightarrow \frac{6x^3-9x^2-1}{2x-3}$$
$$f(x)\cdot g(x) = 3x^2\left(\frac{1}{2x-3}\right) \Rightarrow \frac{3x^2}{2x-3}$$

b)
$$f(x) = x^3+3x, g(x) = 3x^2+1$$
$$f(x)+g(x) = \left(x^3+3x\right)+\left(3x^2+1\right) \Rightarrow x^3+3x^2+3x+1$$
$$f(x)-g(x) = \left(x^3+3x\right)-\left(3x^2+1\right) \Rightarrow x^3+3x-3x^2-1 \Rightarrow x^3-3x^2+3x-1$$
$$f(x)\cdot g(x) = \left(x^3+3x\right)\left(3x^2+1\right) \Rightarrow 3x^5+x^3+9x^3+3x \Rightarrow 3x^5+10x^3+3x$$

c)
$$f(x) = 2x^3-x+5, g(x) = x^2+x+2$$
$$f(x)+g(x) = \left(2x^3-x+5\right)+\left(x^2+x+2\right) \Rightarrow 2x^3+x^2+7$$
$$f(x)-g(x) = \left(2x^3-x+5\right)-\left(x^2+x+2\right) \Rightarrow 2x^3-x^2-2x+3$$
$$f(x)\cdot g(x) = \left(2x^3-x+5\right)\left(x^2+x+2\right) \Rightarrow 2x^5+2x^4+4x^3-x^3-x^2-2x+5x^2+5x+10 \Rightarrow$$
$$2x^5+2x^4+3x^3+4x^2+3x+10$$

4. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ where $(f \circ g)(x) = f(g(x))$

$$f(x) = 2x^2 + 5, g(x) = 4 - 7x$$

a) $(f \circ g)x = 2(4-7x)^2 + 5 \Rightarrow 2(16 - 56x + 49x^2) + 5 \Rightarrow 49x^2 - 112x + 37$

$(g \circ f)x = 4 - 7(2x^2 + 5) \Rightarrow 4 - 14x^2 - 35 \Rightarrow -14x^2 - 31$

$$f(x) = \sqrt{2x+1}, g(x) = x^2 + 3$$

b) b) $(f \circ g)x = \sqrt{2(x^2+3)+1} \Rightarrow \sqrt{2x^2 + 6 + 1} \Rightarrow \sqrt{2x^2 + 7}$

$(g \circ f)x = \left(\sqrt{2x+1}\right)^2 + 3 \Rightarrow 2x + 1 + 3 \Rightarrow 2x + 4$

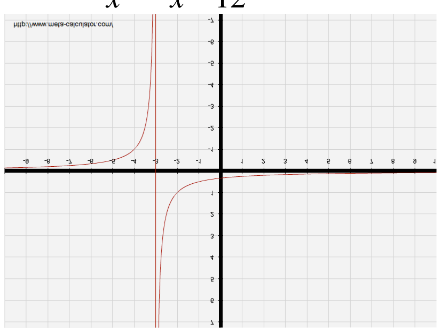
$$f(x) = 2x - 3, g(x) = \frac{x+3}{2}$$

c) $(f \circ g)x = 2\left(\frac{x+3}{2}\right) - 3 \Rightarrow x + 3 - 3 \Rightarrow x$

$(g \circ f)x = \frac{(2x-3)+3}{2} \Rightarrow \frac{2x-3+3}{2} \Rightarrow x$

5. Graph the following functions and determine the value of y as x gets close to the indicated value.

a) $f(x) = \frac{x-4}{x^2 - x - 12}$ as x gets close to 4

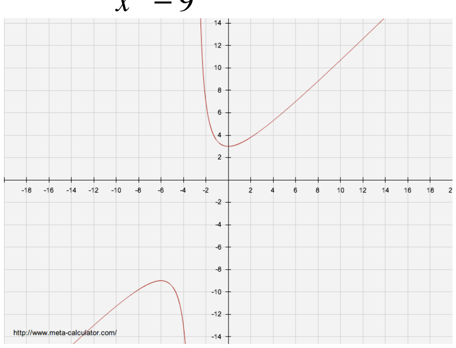


if by direct substitution

$$y = \frac{x-4}{x^2 - x - 12} \Rightarrow \frac{4-4}{4^2 - 4 - 12} = \frac{0}{0} = undefined$$

$y = 0.1424$ approximation from the graph

b) $f(x) = \frac{x^3 - 27}{x^2 - 9}$ as x gets close to 3

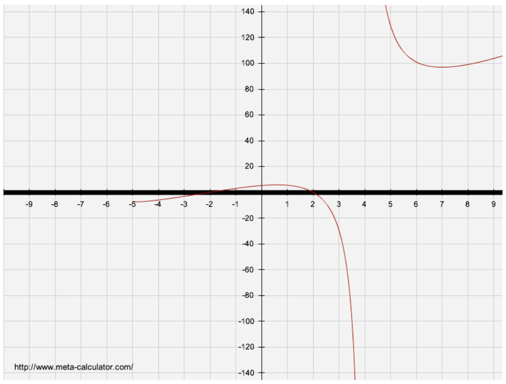


if by direct substitution

$$y = \frac{x^3 - 27}{x^2 - 9} \Rightarrow \frac{3^3 - 27}{3^2 - 9} = \frac{0}{0} = undefined$$

$y = 4.4917$ approximation from the graph

c) $f(x) = \dfrac{4-x^2}{3-\sqrt{x\;\;+5}}$ as x gets close to 2

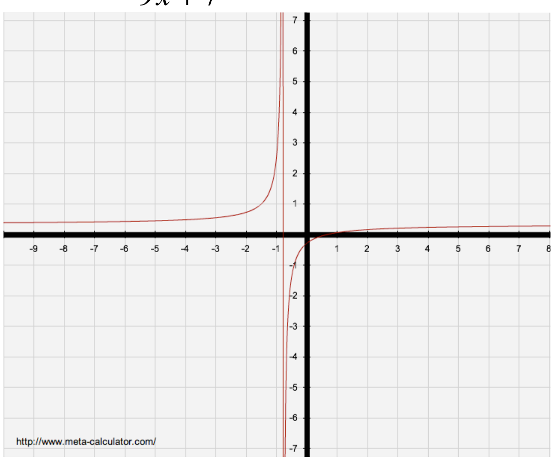


if by direct substitution

$$y = \frac{4-x^2}{3-\sqrt{x+5}} \Rightarrow \frac{4-0^2}{3-\sqrt{0+5}} = \frac{4}{3-\sqrt{5}}$$

$y = 0$ approximation from the graph

d) $f(x) = \dfrac{3x-2}{9x+7}$ as x gets close to ∞

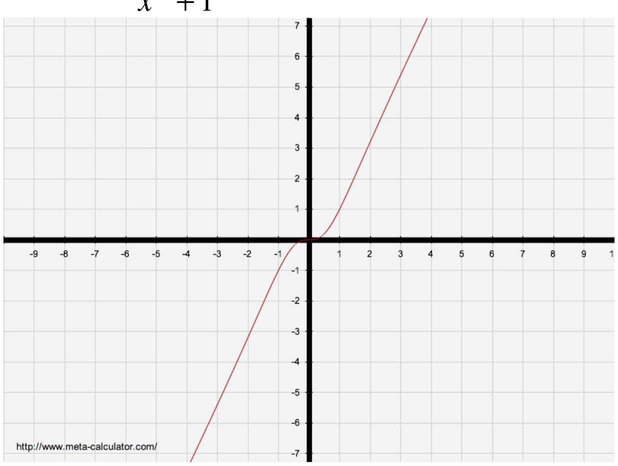


if by direct substitution

$$y = \frac{3x-2}{9x+7} \Rightarrow \frac{3(\infty)}{9(\infty)+7} = ?$$

$y = .33$ approximation from the graph

e) $f(x) = \dfrac{2x^3}{x^2+1}$ as x gets close to ∞



if by direct substitution

$$y = \frac{2x^3}{x^2+1} \Rightarrow \frac{2(\infty)^3}{(\infty)^2+1} = ?$$

$y = \infty$ approximation from the graph